**An Efficient Method** with Tunable Accuracy **for Estimating Expected Interruption Cost of Distribution Systems**

Abstract— The paper presents a novel efficient method with tunable accuracy for estimating expected interruption cost (ECOST) of distribution systems. ECOST index quantifies the reliability of a distribution system in monetary basis. The performance of ECOST estimation could be influenced by various factors such as time-varying load and cost models, computational limitation and random nature of component failure and repair time. Generally, the interruption cost is estimated based on an analytical method which does not consider the input and parameter uncertainties that are represented as random variables. The simulation method based on Monte Carlo (MC) simulation could provide a more accurate approximation of ECOST due to consideration of stochastic factors. However, one basic challenge related to the MC method is the high computational cost in order to run a large number of iterations for a specified high accuracy. Speed up the accurate estimation process using fast computation method could be an important feature in distribution systems management software. An advancement of the MC method with controllable accuracy is the Multilevel Monte Carlo (MLMC) estimator which is proposed to estimate the system ECOST. The proposed method could reduce the huge computational cost needed for accurately estimating the index. To illustrate the performance of this method, five different size distribution systems of Roy Billinton Test System are utilized. The impacts of the network topology, customer load type, time-varying load and cost models, failure and repair statistics on the MLMC based system ECOST assessment performance are also investigated.

Keywords— expected interruption cost (ECOST); computation speedup; Multilevel Monte Carlo simulation; Milstein method.

1. Introduction

Expected interruption cost (ECOST) estimation is an important part of reliability analysis of distribution systems [1]. Knowing the value of a distribution system ECOST in monetary basis could play a significant role for making an optimal investment decision and for deciding the supply of which regions or sectors should be cut off during electricity shortage [2]. ECOST is a completely random variable [3] due to the dependency of random frequency and duration of interruption, time-varying load and cost levels of different customer sectors. In ECOST estimation, interruption frequency and duration are the primary variables. Load level of a given customer sector varies through the duration of the failure period. Similarly, the interruption cost for a given type of customer which is found by analysing Sector Customer Damage Function (SCDF) [4, 5] and varies with the failure duration and starting time. Therefore, analytical method based on average interruption frequency, load and cost models could be replaced by simulation approach which could provide the more accurate result of ECOST estimation by considering random factors [6, 7]. Through the simulation method, information of probability distribution of ECOST index could be obtained which is necessary for distribution systems expansion and future planning.

Simulation approach generally used in distribution systems reliability evaluation [8-13] including ECOST estimation is based on standard Monte Carlo (MC) simulation. MC method generates the stochastic behavior of components outage and repair times. It can be simulated in either sequential or non-sequential mode [14]. In the non-sequential mode, the states of all components are sampled and a non-chronological system state is obtained [15]. On the other hand, the up and down cycles of all components are simulated in the sequential approach and overall system operating cycle is obtained by combining all the component cycles [15]. The sequential MC mode allows chronological issues to be considered [16]. The state duration sampling approach is generally used to simulate chronological issues which provide time-related reliability indices concerning frequency and duration of load point interruption [17]. However, the slow convergence rate of the sequential MC makes it very time consuming when a large number of samples are necessary to obtain a precise result depending on the accuracy level and a number of variables in the system; particularly it is not efficient at simulating rare events. It is mentioned that the estimation of a distribution system ECOST using any computationally inexpensive method with controllable accuracy has not been reported yet. Therefore, a new sequential Multilevel Monte Carlo (MLMC) simulation method has been applied in this study to overcome the MC based ECOST computational burden.

The MLMC estimator is an advanced MC estimator for performing stochastic simulations. In this method, the computational performance of the basic MC method is increased by maintaining the acceptable accuracy of the simulation. The idea of MLMC is to use a hierarchy of computational meshes (levels) instead of using single time discretization level in MC method. By simulating on multiple levels, MLMC runs most of the simulations on the computationally low-cost coarse levels and few simulations on the computationally expensive finest level. Where in MC method, all the samples are only simulated on the finest level [18]. It is simple to understand that the computational cost on a fine level is higher than on coarse level [19]. Thus most of the computational effort in MLMC method is transferred from the finest level to the coarsest one, leading to substantial computational saving. As the simulations are conducted using multiple approximations, therefore the less accurate estimate on the preceding coarse level can be sequentially corrected by averages of the differences of the estimations of two consecutive levels in the hierarchy. Detailed mathematical proofs of MLMC can be found in [20]. Initially, the method was introduced by Heinrich in the context of parametric integration [21]. The method has been extended further by Giles in the context of stochastic differential equations [18]. Since then the method has been applied for uncertainty quantification in various applications [22-28].

The objectives of this study are to investigate the application of MLMC method on estimating the system ECOST as well as to find the effect of different parameters on the computation performance. In the present study, components are represented by two-state model [29]. The operation history of each component is generated based on stochastic differential equation (SDE). Time-varying load and cost models are developed for each of the load points in the system. Milstein discretization scheme is used for the approximate numerical solution of the SDE. A benchmark Roy Billinton Test System (RBTS) [30] is used to apply the method. The performance of the proposed method is compared with the MC method in terms of computation accuracy and speed-up. The effect of different parameters on the MLMC computation method such as network configuration and load type, time-varying load and cost models, network reinforcement, transformer and line failure rate, drift and volatility values are investigated to provide insight into the variation of the ECOST with different system factors.

The remainder of this paper is organized as follows. Section 2 describes the mathematical explanation of the problem and the MLMC approach. In Section 3, ECOST evaluation methodology has been described. This Section consists of five subsections: generation of component operating history, modelling of load, modelling of per unit interruption cost, modelling of system ECOST, and simulation steps to evaluate the ECOST. In Section 4, we investigate the applicability of the proposed MLMC. We evaluate system ECOST considering different factors affecting the results to show how much the estimation results could be diversified from average value while these factors are considered. Finally, Section 5 concludes the paper.

**2. Proposed approach**

2.1. Problem statement

MC method is a straightforward way for the estimation of expectation arising from the stochastic simulation where ECOST is estimated by averaging over a large number of samples on a single fine grid level [31]. Let be the ECOST for this study and is the expectation or quantity of interest. Also, let be the approximation to . If is the sample of and is the number of independent MC samples. Then, an unbiased MC estimator for is

(1)

where , is the variance of this estimate and the rms error is .

To achieve an accuracy of , it requires samples to be simulated. For an increasing accuracy level, the number of samples also increases. As the samples are run on the finest level, the accuracy of evaluation is sufficiently accurate in MC method. However, the problem associated with the MC method is that huge computational burden is introduced due to large sample size. This motivates to find an alternative method which could speed up the system ECOST estimation in distribution systems planning application by maintaining an adequate level of accuracy. Hence the applicability of the proposed MLMC method is investigated in this study to perform this objective.

## **2.2. MLMC method**

In the MC method, the calculation of system ECOST is carried out on a single fine grid level . is a nonnegative integer. In this level, both accuracy and estimation time of the expectation is very high.

In the MLMC method, same ECOST is evaluated but using multiple parts or levels instead of using only one level . The multiple levels can be divided as . Here and indicate the coarsest and finest discretisation levels, respectively. On each level, expectations are estimated using different sample size in a manner which could reduce the overall variance for a specified error tolerance. Initially, a large number of simulations are run on . The purpose of the next level is to add a correction value that initiates to decrease the bias. Based on this correction value, the expected difference from one level to the next fine level is added, until levelis reached. By this way, the less correct estimate on the coarsest grid is successively corrected by the approximations on the subsequent fine grids and thus thefinest grid accuracy is reached. The MLMC expectation can be expressed as:

(2)

Now the estimator for the above MLMC expectation is

(3)

where is an unbiased estimator for using samples on and is the estimator for using samples from .

(4)

(5)

is approximated using the difference between the expectation of the coarse and fine level as . and are estimated using two different timestep sizes and , respectively [32]. On level , a large and fixed value timestep is used to generate a large sample size . This means that the coarsest level is the computationally cheapest level and the simulation time increases with the increase of level. The statistical properties of are unchanged whether the estimation of from level to or from to . This means that both and have the same expected value i.e. . The convergence condition of the proposed method is the target rms error which could be written as follows:

(6)

To obtain an overall mean square error , both the variance and weak error of MLMC estimator could be reduced below . The variance could be reduced by setting a different number of samples at different levels. As the level moves from the initial coarsest level to next finer level, the choice of an optimal is made as follows [18].

(7)

where the cost to compute one sample on the level is for a constant and some . is the rate of computation cost increase with the level . For weak convergence, the test tries to confirm that . Consider, the convergence rate of with for constant is measured by a positive value , i.e., [18]. The remaining error is and the target convergence criterion is . is assumed as the convergence rate of variance with for a constant i.e., For a constant , computational complexity of the MLMC estimator is bounded as [18]:

# (8)

**3. Methodology**

## **3.1. Generation of operating history**

For generating the operating history of any component, the stochastic model of component Time-to-Failure (TTF) is first developed. Consider and are the failure rate and repair time of a component , respectively. Also, consider the SDE of TTF is driven by the Brownian motion [33]. If is the TTF of an event at a time , then SDE of TTF with defined drift , volatility and initial TTF can be modelled using the Brownian motion on the whole time interval [] [34] as follows:

(9)

In this paper, the SDE is solved by Milstein discretisation scheme [32]. The discretisation scheme with time-steps, step size and Brownian increments could be written as:

(10)

where are the normally distributed independent random variables. and . Using Eqn. (10), the operating history of component , could be generated as follows:

(11)

where is a uniformly distributed random variable between [0, 1].

## **3.2. Modelling of load**

Usually load level of a specific customer type fluctuates due to the discrepancy between the hourly consumption levels. In addition, seasonal inconsistency in the weather contributes prominently to loading level diversity [35]. Evaluation of interruption cost based on average load level without considering time-varying diversity factors does not reflect the time-varying nature of system ECOST. Thus accurate approximation of ECOST needs the consideration of modelling of loads throughout a 24-hour period depending on seasons. However, the most utilities collect load data for a specific distribution area only and also individual load point data throughout the daily 24 hours in a year are not usually available. The global behavior of each load type is therefore established by considering typical load consumption cycles which is found as an acceptable method in power systems reliability evaluation study performed by Billinton and Allan [36]. Complete load data of winter season in a weekday for modelling time-varying load models are displayed in Appendix A.

For modelling time-varying load of a load point at an hour , annual peak load , weekly peak load as a percentage of annual peak , daily peak load as a percentage of weekly peak and hourly peak load as a percentage of daily peak are formulated as [36]:

, MW (12)

Consider, the interruption of load point starts and ends at and hours, respectively. Then the average time-varying load level of this load point is evaluated as follows:

, MW (13)

## **3.3. Modelling of per unit interruption cost**

The interruption cost of a load point for any duration is found from SCDF [5]. The cost of a load point per unit interruption depends on the type of the customer connected in that point. The SCDF presents the customer interruption costs as a function of interruption duration. The SCDF for different types of customers are provided in Appendix A. It can be seen that per unit interruption costs for various customer sectors are different by depending on interruption duration. For example, when interruption lasts 1 hour, the maximum and minimum per unit cost are found for office buildings and residential customers, respectively. A linear interpretation of the cost data is used in this study where the interruption duration lies between two separate times.

Based on average cost model ( from SCDF, the interruption cost related to a load point failure for a duration can be expressed as: Here is the customer interruption cost related to a load point . From SCDF, only the average monetary losses of customer interruptions are found. On the other hand, for modelling interruption cost at an hour based on time-varying cost model, the multiplication of from SCDF and time-varying weight factor is used, i.e. . Then average time-varying cost level of a load point for above failure period can be formulated as:

, ($/kW) (14)

## **3.4. Modelling of system ECOST**

For a component failure , the value of average outage rate could be calculated using the following expression:

(f/yr) (15)

where is the number of times component fails during whole simulation period and is the desired number of simulated periods.

For load point , average outage rate is evaluated as follows by accumulating the outage rate of all the failure events connected to this load point.

(f/yr) (16)

where denotes the number of outage events interrupting the service of the load point . Using Eqns. (13), (14) and (16), overall distribution system ECOST can be evaluated as follows.

(k$/yr) (17)

where is the total number of supply points in the system.

## **3.5. Simulation process**

In the simulation, there are two phases. In the 1st phase, the stochastic model of ECOST is established on both coarse and fine levels. In the 2nd phase, overall MLMC estimator is calculated through satisfying the target convergence criteria of the simulation. In the 1st phase, initially, failure rate, repair/switching time of each distribution system component are defined. Additionally, the values of sample size for convergence test (, initial sample size on each level (, drift, volatility and target accuracy level are defined. The model of a component is represented by up-down states. The operating history of each component is generated according to the exponential probability distribution using Eqn. (11). Using Eqn. (13), time-varying load model of each load point during failure period is established based on peak load, hourly, daily and weekly load diversity factors. Similarly, time-varying cost model of each load point is established based on SCDF and cost weight factors by following Eqn. (14). After this, the value of each component average failure rate is calculated using Eqn. (15). The value of each load point average failure rate is calculated by accumulating the individual component value connected to the relevant load point by following Eqn. (16). System ECOST is then computed using Eqn. (17). A flowchart of the ECOST estimation on coarse and fine levels is shown in Fig. 1(a).

 

(a) (b)

Fig. 1. Flowchart (a) ECOST estimation on coarse and fine levels (b) Convergence test

In the 2nd phase, initially, the finest grid level of simulation is set at . The number of samples on each level is then determined using initial sample size. The sum of ECOST values on coarse and fine levels is simultaneously updated. Then, the absolute average value of the index and variance are calculated on each level. The optimal sample size on each level is determined based on Eqn. (7). Next, the optimal sample size on each level is compared to the already computed on this level. If the is larger than , then the additional samples on each level are evaluated and the values of mean, variance on each level are also updated. The purpose of determining the optimal is to make the variance term of Eqn. (6) smaller than . The test for the weak convergence is then performed which ensures the remaining bias error < . If the bias error remains greater than , then the finest level is reset as . The entire process is repeated again until the target accuracy level is achieved. Finally, the combined multilevel estimator for system ECOST is computed using Eqn. (3). A flowchart for convergence test is presented in Fig. 1(b).

**4. Test systems and simulation results**

## **4.1. Test Systems**

Five load busbars of a six-busbar test system-RBTS are used as test distribution systems. Detailed diagrams of the distribution systems at buses 2-6 in the RBTS are found in Appendix B. Basic data of the distribution systems is presented in Table 1. The customer data, load data and types, feeder section length data and component reliability data are taken from [30], [37]. The availability and reliability of breakers, fuses and disconnecting switches are considered as 100% in all the test systems. The availability of alternative supply source is also considered for all systems. The service of failed low voltage transformers is generally restored by repairing rather than replacing.

A target accuracy level of =3% is used to approximate the ECOST of the distribution systems. For all the systems, setting parameters for the MLMC simulation are: =5000, =500, =0.01 and =0.8. The methodology is implemented using MATLAB and all computations are performed using an Intel Core i7-4790 3.60-GHz processor.

Table 1. Basic data of test distribution systems

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Distribution  system | Typical type | Customers types | No of LPs | Average load  (MW) |
| B2 | Urban | R, GI, SI, C | 22 | 12.291 |
| B3 | Large user | LU, SI, C, R, OB | 44 | 52.63 |
| B4 | Complex urban | R, SI, C | 38 | 24.58 |
| B5 | Urban | R, GI, OB, C | 26 | 11.29 |
| B6 | Rural | A, SI, C, LU | 40 | 10.7155 |
| A-Agricultural, C-Commercial, GI-Government and Institutional, R-Residential, LU-Large users, OB-Office buildings, SI-Small industrial, LPs-Load points | | | | |

## **4.2. Simulation results**

## **4.2.1. Effect of network configuration and load type**

Table 2 presents the effect of network configuration and load types in ECOST variation using MC and MLMC methods based computation. The RBTS distribution systems connected to five load buses (B2 to B6) are considered for this purpose [30]. By comparing different test systems, it is seen that the maximum and minimum system ECOST values are found in B3 and B6 systems, respectively. In Eqn. (17), we find that the amount of system ECOST depends on the failure rate, load level and interruption cost of the interrupted load points. In the B3 system, there are five types of loads such as residential, large users, small industrial, commercial and office building users and the total amount of average load for all 44 load points is 52.63 MW. On the other hand, in the B6 system, there are four types of loads such as residential, small industrial, commercial and agricultural and the total amount of average load for all 40 load points is 10.7155 MW. In most of the cases, average load level per load point in the B3 system is higher than the B6 system. Due to having load points with high interruption cost and duration in B3, it gives the large value of ECOST than the B6 system.

More specifically, the availability of commercial load highly increases interruption cost for all the systems except the B6 system, although the peak load level of this load type is not maximum. This happens due to the large amount of per unit interruption cost for commercial load type as displayed in SCDF [5]. In the B6 system, there are a large number of residential load points, where only two commercial load points are connected to the system.

The magnitude of ECOST also varies with the network topologies and loading types. Interruptions in different load connected systems have very different consequences. A test system with smaller number of load points, but having supplies to commercial customers (due to huge interruption cost) may increase the ECOST value in the peak period.

For validation, the results obtained from the proposed method should agree with the results from the analytical method. E[MC] and E[MLMC] are the percentages of the difference of ECOST values using MC and MLMC methods with respect to the analytical value. The results show that the ECOST values using MLMC method are very close to the values from MC and analytical methods. The absolute value of maximum E[MLMC] is 3.25% for B2 system. These results are generally acceptable for an application with uncertainty quantifications. This proves the accuracy of the proposed MLMC approach.

As displayed in Table 2, the maximum and minimum computation times are required for the distribution systems connected to B3 and B6, respectively. In all cases, the percentage of computation speedup is above 90%. For example, the proposed and MC methods need 1.85 and 49.33 seconds, respectively for the B4 system. The MLMC method improves the calculation efficiency of the MC simulation by reducing the number of iterations on the finest level. For example, the proposed method requires 3293 iterations on the finest level and the MC method needs about 19000 iterations for target convergence of ECOST estimation. Due to a large number of required samples, MC method provides ECOST with noticeably high accuracy compared to the MLMC method as shown in Table 2.

Table 2. ECOST using average load and cost models

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| System | MC  (k$/year) | MLMC  (k$/year) | MC  (s) | MLMC  (s) | E[MC]  (%) | E[MLMC]  (%) | Speedup  (%) |
| B2 | 185.51 | 190.18 | 35.23 | 1.14 | -0.72 | -3.25 | 96.76 |
| B3 | 267.52 | 274.73 | 69.27 | 2.74 | 0.99 | -1.67 | 94.73 |
| B4 | 225.66 | 231.81 | 49.33 | 1.85 | 1.16 | -1.52 | 96.24 |
| B5 | 222.84 | 228.74 | 47.40 | 1.58 | 1.32 | -1.29 | 96.67 |
| B6 | 132.12 | 135.63 | 27.06 | 0.9 | -0.03 | -2.68 | 96.67 |

## **4.2.3. Effect of network reinforcement**

More than 80% of the customer interruptions happen due to the fault in the distribution systems. Availability of different protective and switching equipment could reduce the number and duration of these interruptions and increase the system reliability, i.e. more investment on utility could reduce the interruption cost. The variation of ECOST values and their estimation time demonstrate the effect of network reinforcement on cost value and estimation time. Six case studies are carried out as Table 3, where availability of protective devices and switches are considered in various combinations for the B2 system. The maximum and minimum ECOST values are found in case B and E, respectively. In case E, the availability of switches, fuses, alternative supply is considered with the restoration of low-voltage transformer action by replacement. On the other hand, in case B, all these protective equipment are unavailable with transformer action restoration by time-consuming repairing. In fact, the more investment in the protective equipment reduces the interruption effect and as a result, the value of ECOST is also reduced.

Table 4 shows the computational performance of the MC and MLMC methods for all case studies. By comparing with the MC based estimation, the proposed method can estimate ECOST with an acceptable accuracy and the proposed method is considerably more efficient than standard MC. The maximum and minimum simulation times are required for case B and E, respectively. In these cases, the percentages of time-saving using proposed method are 95.66% and 96.52%, respectively. It can be concluded that the magnitude and computation time of ECOST are changed for both methods by the reinforcement.

Table 3. Cases for network reinforcement effect analysis

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Case | Disconnecting Switches | Fuses | Alternative Supply | Transformer Action Restoration |
| A | Yes | Yes | Yes | Repairing |
| B | No | No | No | Repairing |
| C | No | Yes | No | Repairing |
| D | Yes | No | Yes | Repairing |
| E | Yes | Yes | Yes | Replacement |
| F | Yes | No | No | Repairing |

Table 4. ECOST computation performance variation for network reinforcement

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Case | ECOST (k$/yr) | | CPU time (s) | |
| MC | MLMC | MC | MLMC |
| A | 185.51 | 190.18 | 35.23 | 1.14 |
| B | 1253.90 | 1287.50 | 1110.13 | 48.19 |
| C | 220.16 | 225.95 | 49.67 | 1.71 |
| D | 299.28 | 307.37 | 80.19 | 2.76 |
| E | 28.60 | 29.33 | 0.9151 | 0.03228 |
| F | 907.76 | 931.79 | 317.25 | 25.15 |

## **4.2.4. Effect of transformer failure rate**

Customer interruption cost could be reduced by decreasing the failure rate of the low-voltage distribution transformer (TFR). In order to examine the effect of this parameter, TFR is variedfrom 0.005 f/yr to 0.25 f/yr for ECOST calculation of B2 and B6 systems. From Fig. 4, it is seen that the system ECOST increases gradually with the increase of TFR. For the B2 system, the rate of ECOST increase is higher than the B6 system. This is mainly due to higher interruption cost of affected load points in the B2 system. From the computational perspective, the calculation time of ECOST increases also as TFR is increased and the proposed method could save a huge amount of CPU time compared with direct MC method as shown in Fig. 5.

Fig. 4. ECOST magnitude variation for varying transformer failure rate

Fig. 5. ECOST computation time variation for varying transformer failure rate

## **4.2.5. Effect of line failure rate**

The failure rate of overhead distribution line (LFR) has an impact of interruption cost estimation. For investigating the effect, a sensitivity analysis is carried out where LFR is variedfrom 0.025 f/yr to 0.15 f/yr for ECOST calculation of B2 and B6 systems. As shown in Fig. 6, the system ECOST increases at a higher rate with the increase of LFR. Overhead line is a very basic component of a feeder. In the radial system, any failure in a feeder line section could interrupt the function of all the connected supply points of the feeder. From the computational perspective in Fig. 7, the calculation time of ECOST increases also as LFR is increased and the proposed method could accelerate the computation process compared with MC method. Similarly, the length of a line section influences the ECOST estimation since a long-length line increases the failure rate compared with the short-length line.

Fig. 6. ECOST variation for varying line failure rate

Fig. 7. ECOST computation time for varying line failure rate

## **4.2.6. Effect of drift and volatility values**

The values of drift and volatility parameters of stochastic failure process could be found from time series of TTF [38]. However, due to the data constraint, the drift and volatility in this study are determined by adjusting based on the accuracy and assumed these values as constant for all case studies. Here the effect of both parameters on system ECOST evaluation accuracy and time are investigated. For example, Table 5 presents the variation of ECOST value and computation time of B4 system by varying while 0.8 is set as constant. A gradual decrease of ECOST value and its computation time is noticed when is varied from 0.03 to 0.2. Similarly, Table 6 shows the variation of ECOST value and computation time of the same system by varying while 0.8 is set as constant. The results show that both the computation time and value of ECOST increase slowly with the increase of .

Table 5. ECOST variation for varying drift

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | 0.03 | 0.05 | 0.09 | 0.12 | 0.2 |
| ECOST (k$/year) | 227.11 | 222.72 | 214.14 | 208.23 | 193.85 |
| CPU time (s) | 1.73 | 1.65 | 1.40 | 1.30 | 1.02 |

Table 6. ECOST variation for varying volatility

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | 0.3 | 0.5 | 1.0 | 1.20 | 1.25 |
| ECOST (k$/year) | 228.80 | 228.90 | 234.81 | 239.47 | 240.90 |
| CPU time (s) | 0.58 | 0.85 | 3.25 | 5.84 | 6.72 |

## **4.2.7. Effect of accuracy level**

The proposed method could be more efficient for estimating small probabilities, i.e., the probabilities of rare events, with a high accuracy when compared to MC method. For example, Table 7 presents the performance of B4 system ECOST at different accuracy levels. It is found that MLMC method greatly reduces the computation cost without compromising the accuracy of the index.

Table 7. ECOST computation at different accuracy levels

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | MC (k$/year) | MLMC (k$/year) | MC (s) | MLMC (s) |
| 0.001 | 225.67 | 231.68 | 38812.93 | 1641.74 |
| 0.01 | 225.69 | 231.73 | 359.43 | 16.83 |
| 0.03 | 225.66 | 231.81 | 49.33 | 1.85 |
| 0.05 | 225.72 | 231.44 | 18.64 | 0.68 |

**5. Conclusions**

This paper briefly illustrates the application of a novel Multilevel Monte Carlo (MLMC) method in distribution systems interruption cost estimation. MLMC is a variance reduction technique of Monte Carlo (MC) simulation. MLMC reduces the computational cost by performing most of the simulations with low accuracy at a correspondingly low cost on the coarse grids and relatively few simulations being performed on the computationally expensive fine grids at high accuracy and high cost. Both accuracy and computational efficiency of the MLMC are considered to demonstrate the practicability of the proposed method. Comparisons of results obtained using the basic MC simulation are presented. The results show that the proposed method could estimate the average customer interruption cost accurately and also gives a significant speed-up over the MC based computation.

A number of sensitivity analyses have been carried out to show the effect of different parameters on the computation process. We found that network configuration and load type, time-varying load and cost models, network reinforcement, transformer and line failure rate, drift and volatility values parameters have a considerable effect of MLMC based system ECOST computation.

Due to the data constraint, the research on comparisons with other advanced MC methods and the probability distributions of MLMC [39] based ECOST estimations will be carried out in the future. For estimating MLMC Maximum Entropy method based PDF in the current study, the authors believe that more studies should be conducted where some additional algorithms, coding, methodology, sensitivity studies of some assumptions (such as number of moments in the Maximum Entropy method) need to be completed for ECOST analysis in order to publish the PDF. Therefore, in the present publication, we set our main focus to provide the MLMC based ECOST estimation to show its speed-up capability and also to present some sensitivity analysis which is necessary to analyse the large real-life systems. The computationally efficient algorithm and simulation examples presented in this paper would potentially help system planners to collect the useful information of reliability cost of their respective distribution systems. We believe that the proposed algorithm will be able to speed up the decision-making process in reliability improvement process.

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**Appendix A**

Load [5, 36, 40] and SCDF data [5]

Fig. A.1. Hourly load weight factors for different customer sectors

Table A.3. SCDF for different customer types

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| User sector | Interruption duration (Min.) & cost($/kW) | | | | |
|  | 1 min. | 20 min. | 60 min. | 240 min. | 480 min. |
| Large user | 1.005 | 1.508 | 2.225 | 3.968 | 8.240 |
| Industrial | 1.625 | 3.868 | 9.085 | 25.16 | 55.81 |
| Commercial | 0.381 | 2.969 | 8.552 | 31.32 | 83.01 |
| Residential | 0.001 | 0.093 | 0.482 | 4.914 | 15.69 |
| Govt./Inst. | 0.044 | 0.369 | 1.492 | 6.558 | 26.04 |
| Office buildings | 4.778 | 9.878 | 21.06 | 68.83 | 119.2 |
| Agricultural | 0.060 | 0.343 | 0.649 | 2.064 | 4.120 |

Fig. A.2. Hourly cost weight factors for different customer sectors

**Appendix B**

Single line diagrams of distribution systems [30]

  

 

Fig. B.1. Distribution systems for RBTS (a) Bus 2 (b) Bus 3 (c) Bus 4 (d) Bus 5 and (e) Bus 5